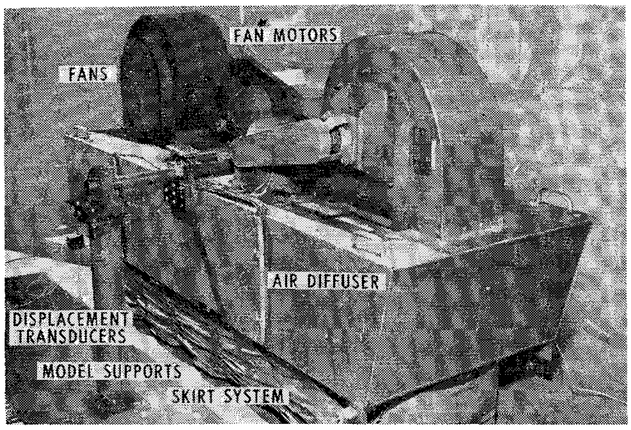

Fig. 3 ACV pitch test assembly.

and heave. As a result, the skirt leakage pressure-flow relation is represented by

$$Q_L = CS(h \pm X\theta - Z)(2\Delta P/\rho)^{1/2} \qquad (2)$$

The plenum pneumatic capacitances are based on isentropic compressibility. The pressure rates associated with the inner circular skirt plenums and the inskirt plenum, which lies between the inner circular skirts and the outer peripheral skirt, vary with vehicle motion in pitch and heave where

$$dP/dt = (k\bar{P}/V)[\Sigma Q + A(dZ/dt \pm xd\theta/dt)] \qquad (3)$$

The equations of vehicle motion in pitch and heave represent a rigid craft operating over a nondeforming surface with zero forward speed and are simplified by limiting the motion to small perturbations. A body-fixed coordinate system is used with the origin fixed at the vehicle center of gravity. A further simplification is realized due to symmetry about the x-z and y-z planes of the vehicle. The equation of motion in heave for the multiple-skirt ACV is

$$d^2z/dt^2 = g - (g/w)T(t) - (g/w)(\Sigma A_0P_P + \Sigma A_JP_J + A_LP_L) \qquad (4)$$

The equation of motion in pitch is given by

$$d^2\theta/dt^2 = [\Sigma A_JP_JX + X_TT(t)]/I_y \qquad (5)$$

The air cushion pressures contained in the vehicle motion equations are obtained from the nonlinear air cushion supply system model. The resulting forces and moments are coupled to vehicle motion in a feedback manner since the air flow in the skirted regions beneath the vehicle is modulated by vehicle motions in pitch and heave.

Analog Computer Simulation

The equations for the behavior of the multiple-skirt ACV are programed on a PACE 231R analog computer. The use of the computer simulation results in predictions of the steady-state trim and the transient variation in the pressures, flow rates, and motions of the air cushion vehicle for transient excitations. The transient excitations are in the form of step changes in applied force for heave and applied moment for pitch. Computer results were obtained for transient motion of an ACV with a weight of 375 lb, a pitch moment of inertia of 40 ft-lb-sec^2, a length to beam ratio of 2.7, and a cushion density of 7.2 lb/ft^3. The results are shown in Fig. 2 and indicate lightly damped dynamic behavior in pitch and heave. In particular, the predicted damping ratios are 0.08 for heave and 0.015 for pitch.

Experimental Check of the Model and Conclusions

To verify the analytical predictions, the multiple-skirt ACV is mounted in a pitch test fixture with hovergap adjustability as shown in Fig. 3 and subjected to step changes in an externally applied moment during cushion operation. The vehicle pitch motion is sensed by a pull wire displacement transducer connected between the vehicle and the level concrete pad beneath the vehicle. The oscillatory pitch motion output from this sensor is recorded using a galvanometer recorder and is shown in Fig. 2.

An additional pitch transient test is run with a calibrated spring at each corner of the ACV with the air cushion circuit inactive. From this test, the rotational supporting structure damping ratio is found to be 0.004 and the vehicle pitch moment of inertia is found to be 31.0 ft-lb-sec^2.

At a 0.6-in. hovergap corresponding to the vehicle trim conditions in the computer study and correcting for supporting structure damping and the moment of inertia difference, the pitch damping and natural frequency obtained from the vehicle test data agree very closely with those obtained from the analog computer predictions. As a result, the analytical technique described in this Note is a valid design tool for predicting the overland dynamics of a multiple-skirt ACV.

Gross Thrust Coefficient—Turbofan Engines

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Nomenclature

A	=	exhaust nozzle area, ft^2
Cg	=	exhaust nozzle gross thrust coefficient
Fg	=	gross thrust, lb
Mn	=	flight Mach number
NPR	=	nozzle pressure ratio, P_t/P_a
P_a	=	ambient pressure, in. HgA
P_t	=	nozzle exit total pressure, in. HgA
SLS	=	sea level static
γ	=	ratio of specific heats

Introduction

CURRENT and future military aircraft will utilize turbojet and turbofan engines as their basic powerplants. In developing a new-model engine, the engine manufacturer issues an engine model specification that contains performance estimates for the range of altitudes and Mach numbers over which the engine can be used. These estimates enable the airframe manufacturer to determine installed engine performance and the Government to determine if the proposed engine will be adequate for specific mission requirements.

The estimation of gross thrust at a given flight condition is of primary importance. This can be done by calculating an ideal engine gross thrust and multiplying by an appropriate nozzle gross thrust coefficient (Cg). This Note concerns the gross thrust coefficient to be used for thrust estimates on turbofan engines.

Current Method of Estimating Thrust

The current method used for both turbojet and turbofan engines is to calculate an ideal gross thrust and then multiply by the proper value of Cg ($Fg = Cg \cdot Fg_{\text{ideal}}$). Figure 1 illustrates schematically the exhaust nozzle of a turbofan engine. The gross thrust coefficient Cg is defined as the ratio of actual to ideal gross thrust, with the ideal gross thrust based on an isentropic expansion of the pressure ratio P_t/P_a across the nozzle. Since the engine manufacturers do not have the

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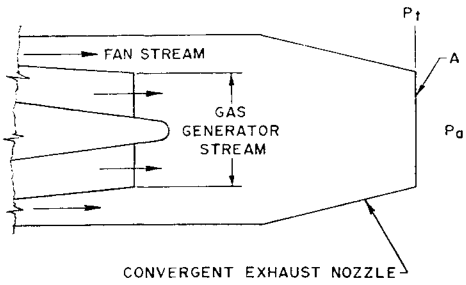

$$Cg = \frac{Fg\ \text{actual}}{Fg\ \text{ideal}}$$

where

$$Fg\ \text{ideal} = \frac{141.454\,\gamma\,Pa\,A}{\gamma - 1}\left[\left(\frac{Pt}{Pa}\right)^{\frac{\gamma-1}{\gamma}} - 1\right] \quad \text{Subcritical Flow}$$

and

$$Fg\ \text{ideal} = 70.727\,A\left\{\left[\frac{\gamma+1}{\left(\frac{\gamma+1}{2}\right)^{\frac{\gamma}{\gamma-1}}}\right]Pt - Pa\right\} \quad \text{Critical Flow}$$

Fig. 1 Turbofan engine exhaust nozzle gross thrust coefficient.

facilities to simulate the complete range of altitudes and Mach numbers of the engine envelope and cannot determine the actual engine thrust, an alternate method is used to obtain the *Cg* relationship.

Usually, model tests are run over a wide range of nozzle pressure ratios with a convergent nozzle expanding a single gas stream. Actual thrust is determined, an ideal thrust is calculated, and a *Cg* determined (Fig. 2). Full-scale engine tests are run at sea level static conditions and these results plotted and compared with the model test data. As shown on Fig. 2, the full-scale SLS data extend only out to a nozzle pressure ratio of 2. This *Cg* relationship is then extrapolated to higher nozzle pressure ratios, and is used in conjunction with the calculated ideal gross thrust. For turbojet engines, the *Cg* relationship is a unique function of nozzle pressure ratio, and has been used satisfactorily to estimate the thrust at a given flight condition. In the case of turbofan engines, however, the method is subject to question.

Nonuniqueness of Turbofan Engine *Cg*

Engine tests at the Naval Air Propulsion Test Center (NAPTC) on several turbofan engines have shown that the *Cg* relationship is not a unique function of nozzle pressure ratio. A family of curves occurs at constant nozzle pressure ratios below 3.0 between sea level static and higher Mach number conditions, as shown on Fig. 3. The separation appears to be primarily dependent on changes in flight Mach number, which cause changes in the engine bypass ratio and a redistribution of energy within the engine. The deviation

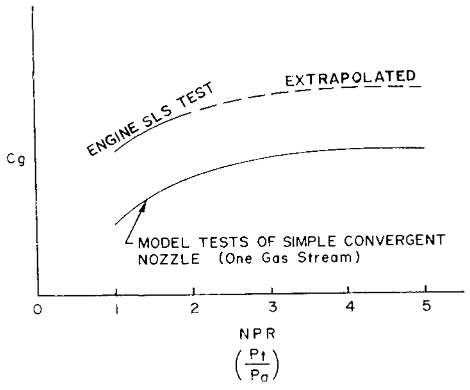

Fig. 2 Current method of determining gross thrust coefficient relationship.

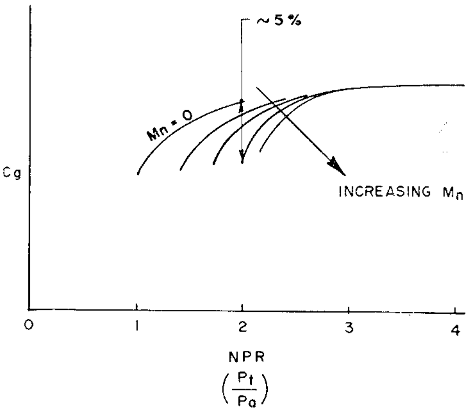

Fig. 3 Gross thrust coefficient relationship for turbofan engines.

in *Cg* between static and high Mach number conditions can be as much as 5%. A comparison with the results of Refs. 1 and 2 indicated this characteristic should be expected for turbofan engines.

Model tests of convergent nozzles with a single gas stream are not sufficient to simulate actual conditions present in the exhaust nozzles of turbofan engines, since the turbofan engine has two coaxial gas streams that discharge into a common nozzle.

Similar results would be expected for turbojet engines with exhaust nozzles that combine secondary air with primary core flow before expansion through the nozzle.

Significance of Results

As shown on Fig. 3, the *Cg* has its highest value at sea level static conditions ($Mn = 0$) for a given nozzle pressure ratio. At higher Mach numbers, it will be lower. Thus, for turbofan engines, the use of a sea level static *Cg* schedule to estimate engine thrust at other flight conditions will result in an optimistic gross thrust estimate at these conditions. A high estimate in thrust will result in a low estimate for specific fuel consumption. The amount of difference in the *Cg* between static and higher Mach number conditions may be critical for a given aircraft mission requirement (e.g., aircraft range at cruise conditions). Consequently, the present method for estimating engine gross thrust needs improvement.

Proposed Improvements

It appears at this time that two approaches can be taken. They are as follows:

1) Conduct model tests that simulate the full-scale nozzle, i.e., two coaxial streams discharging into a common nozzle. Provisions should be made for varying the temperature, pressure, area, and weight-flow ratios of the two streams. Then the engine should be run at several Mach numbers at sea level and used in conjunction with model test results to develop a family of *Cg* curves that will give more accurate thrust estimates at any flight condition.

2) The ideal thrust term in the *Cg* equation should be modified to reflect the presence of two gas streams, with thrust augmentation due to mixing taken into account. This would be an attempt to make the *Cg* relationship also a unique function of nozzle pressure ratio for turbofan engines.

The second approach will be investigated at NAPTC.

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